

Pari-GP reference card

(PARI-GP version 2.15.3)

Note: optional arguments are surrounded by braces {}.

To start the calculator, type its name in the terminal: **gp**

To exit **gp**, type **quit**, **\q**, or **<C-D>** at prompt.

Help

describe function	?function
extended description	??keyword
list of relevant help topics	???pattern
name of GP-1.39 function f in GP-2.*	whatnow(f)

Input/Output

previous result, the result before	%, %`, %`` , etc.
n -th result since startup	% n
separate multiple statements on line	;
extend statement on additional lines	\
extend statements on several lines	{ seq_1 ; seq_2 ;}
comment	/* ... */
one-line comment, rest of line ignored	\\ ...

Metacommands & Defaults

set default d to val	default({ d },{ val })
toggle timer on/off	#
print time for last result	##
print defaults	\d
set debug level to n	\g n
set memory debug level to n	\gm n
set n significant digits / bits	\p n , \pb n
set n terms in series	\ps n
quit GP	\q
print the list of PARI types	\t
print the list of user-defined functions	\u
read file into GP	\r $filename$
set debuglevel for domain D to n	setdebug(D,n)

Debugger / break loop

get out of break loop	break or <C-D>
go up/down n frames	dbg_up({ n }), dbg_down
set break point	breakpoint()
examine object o	dbg_x(o)
current error data	dbg_err()
number of objects on heap and their size	getheap()
total size of objects on PARI stack	getstack()

PARI Types & Input Formats

t_INT . Integers; hex, binary	± 31 ; $\pm 0x1F$, $\pm 0b101$
t_REAL . Reals	± 3.14 , 6.022 E23
t_INTMOD . Integers modulo m	Mod(n,m)
t_FRAC . Rational Numbers	n/m
t_FFELT . Elt in finite field $\mathbf{F}_q$	ffgen(q , 't)
t_COMPLEX . Complex Numbers	$x + y * I$
t_PADIC . p -adic Numbers	$x + O(p^k)$
t_QUAD . Quadratic Numbers	$x + y * \text{quadgen}(D, \{ 'w \})$
t_POLMOD . Polynomials modulo g	Mod(f,g)
t_POL . Polynomials	$a * x^n + \dots + b$
t_SER . Power Series	$f + O(x^k)$
t_RFRAC . Rational Functions	f/g
t_QFB . Binary quadratic form	Qfb(a,b,c)
t_VEC/t_COL . Row/Column Vectors	[x,y,z], [x,y,z]~
t_VEC integer range	[1..10]

t_VECSMALL . Vector of small ints	Vecsmall([x,y,z])
t_MAT . Matrices	[$a,b;c,d$]
t_LIST . Lists	List([x,y,z])
t_STR . Strings	"abc"
t_INFINITY . $\pm\infty$	+oo, -oo

Reserved Variable Names

$\pi \approx 3.14$, $\gamma \approx 0.57$, $C \approx 0.91$, $I = \sqrt{-1}$	Pi, Euler, Catalan, I
Landau's big-oh notation	O

Information about an Object, Precision

PARI type of object x	type(x)
length of x / size of x in memory	# x , sizebyte(x)
real precision / bit precision of x	precision(x), bitprecision(x)
p -adic, series prec. of x	padicprec(x,p), serprec(x,v)
current dynamic precision	getlocalprec, getlocalbitprec

Operators

basic operations	+, -, *, /, ^, sqr
$i \leftarrow i+1$, $i \leftarrow i-1$, $i \leftarrow i*j$, ...	i++, i--, i*=j,...
Euclidean quotient, remainder	$x \backslash y$, $x \backslash y$, $x \% y$, divrem(x,y)
shift x left or right n bits	$x << n$, $x >> n$ or shift($x, \pm n$)
multiply by 2^n	shiftmul(x,n)
comparison operators	<=, <, >=, >, ==, !=, ==, lex, cmp
boolean operators (or, and, not)	, &&, !
bit operations	bitand, bitneg, bitor, bitxor, bitnegimply
maximum/minimum of x and y	max(x,y), min(x,y)
sign of x (gives $-1, 0, 1$)	sign(x)
binary exponent of x	exponent(x)
derivative of f , 2nd derivative, etc.	f' , f'' , ...
differential operator	diffop($f,v,d,\{n=1\}$)
quote operator (formal variable)	'x
assignment	x = value
simultaneous assignment $x \leftarrow v[1]$, $y \leftarrow v[2]$	[x,y] = v

Select Components

Caveat: components start at index $n = 1$.

n -th component of x	component(x,n)
n -th component of vector/list x	$x[n]$
components $a, a+1, \dots, b$ of vector x	$x[a..b]$
(m,n) -th component of matrix x	$x[m,n]$
row m or column n of matrix x	$x[m,]$, $x[,n]$
numerator/denominator of x	numerator(x), denominator(x)

Random Numbers

random integer/prime in $[0,N[$	random(N), randomprime(N)
get/set random seed	getrand, setrand(s)

Conversions

to vector, matrix, vec. of small ints	Col/Vec, Mat, Vecsmall
to list, set, map, string	List, Set, Map, Str
create $(x \bmod y)$	Mod(x,y)
make x a polynomial of v	Pol($x,\{v\}$)
variants of Pol <i>et al.</i> , in reverse order	Polrev, Vecrev, Colrev
make x a power series of v	Ser($x,\{v\}$)
convert x to simplest possible type	simplify(x)
object x with real precision n	precision(x,n)
object x with bit precision n	bitprecision(x,n)
set precision to p digits in dynamic scope	localprec(p)
set precision to p bits in dynamic scope	localbitprec(p)

Character strings

convert to TeX representation	strtex(x)
string from bytes / from format+args	strchr, sprintf
split string / join strings	strsplit, strjoin
convert time t ms. to h, m, s, ms format	strtime(t)
Conjugates and Lifts	
conjugate of a number x	conj(x)
norm of x , product with conjugate	norm(x)
L^p norm of x (L^∞ if no p)	normlp($x,\{p\}$)
square of L^2 norm of x	norml2(x)
lift of x from Mods and p -adics	lift, centerlift(x)
recursive lift	liftall
lift all t_INT and t_PADIC ($\rightarrow$ t_INT)	liftint
lift all t_POLMOD ($\rightarrow$ t_POL)	liftpol

Lists, Sets & Maps

Sets (= row vector with strictly increasing entries w.r.t. cmp)	
intersection of sets x and y	setintersect(x,y)
set of elements in x not belonging to y	setminus(x,y)
symmetric difference $x \Delta y$	setdelta(x,y)
union of sets x and y	setunion(x,y)
does y belong to the set x	setsearch($x,y,\{flag\}$)
set of all $f(x,y)$, $x \in X$, $y \in Y$	setbinop(f,X,Y)
is x a set ?	setisset(x)

Lists. create empty list: $L = \text{List}()$

append x to list L	listput($L,x,\{i\}$)
remove i -th component from list L	listpop($L,\{i\}$)
insert x in list L at position i	listinsert(L,x,i)
sort the list L in place	listsort($L,\{flag\}$)

Maps. create empty dictionary: $M = \text{Map}()$

attach value v to key k	mapput(M,k,v)
recover value attach to key k or error	mapget(M,k)
is key k in the dict? (set v to $M(k)$)	mapisdefined($M,k,\{\&v\}$)
remove k from map domain	mapdelete(M,k)

GP Programming

User functions and closures

x,y are formal parameters; y defaults to Pi if parameter omitted; z,t are local variables (lexical scope), z initialized to 1.

fun(x, y=Pi) = my(z=1, t); seq

fun = (x, y=Pi) -> my(z=1, t); seq

attach help message h to s	addhelp(s,h)
undefine symbol s (also kills help)	kill(s)
Control Statements (X : formal parameter in expression seq)	
if $a \neq 0$, evaluate seq_1 , else seq_2	if($a,\{seq_1\},\{seq_2\}$)
eval. seq for $a \leq X \leq b$	for($X = a,b,seq$)
...for $X \in v$	foreach(v,X,seq)
...for primes $a \leq X \leq b$	forprime($X = a,b,seq$)
...for primes $\equiv a \pmod q$	forprimestep($X = a,b,q,seq$)
...for composites $a \leq X \leq b$	forcomposite($X = a,b,seq$)
...for $a \leq X \leq b$ stepping s	forstep($X = a,b,s,seq$)
...for X dividing n	fordiv(n,X,seq)
... $X = [n, factor(n)]$, $a \leq n \leq b$	forfactored($X = a,b,seq$)
...as above, n squarefree	forsquarefree($X = a,b,seq$)
... $X = [d, factor(d)]$, $d \mid n$	fordivfactored(n,X,seq)
multivariable for, lex ordering	forvec($X = v,seq$)

loop over partitions of n
... permutations of S
... subsets of $\{1, \dots, n\}$
... k -subsets of $\{1, \dots, n\}$
... vectors $v, q(v) \leq B$; $q > 0$
... $H < G$ finite abelian group
evaluate seq until $a \neq 0$
while $a \neq 0$, evaluate seq
exit n innermost enclosing loops
start new iteration of n -th enclosing loop
return x from current subroutine

Exceptions, warnings
raise an exception / warning
type of error message E
try seq_1 , evaluate seq_2 on error

Functions with closure arguments / results
number of arguments of f
select from v according to f
apply f to all entries in v
evaluate $f(a_1, \dots, a_n)$
evaluate $f(\dots f(f(a_1, a_2), a_3) \dots, a_n)$
calling function as closure

Sums & Products
sum $X = a$ to $X = b$, initialized at x
sum entries of vector v
product of all vector entries
sum $expr$ over divisors of n
... assuming $expr$ multiplicative
product $a \leq X \leq b$, initialized at x
product over primes $a \leq X \leq b$

Sorting
sort x by k -th component
min. m of x ($m = x[i]$), max.
does y belong to x , sorted wrt. f
 $\prod g^x \rightarrow$ factorization ($\Rightarrow$ sorted, unique g)

Input/Output
print with/without $\backslash n$, TeX format
pretty print matrix
print fields with separator
formatted printing
write $args$ to file
write x in binary format
read file into GP
... return as vector of lines
... return as vector of strings
read a string from keyboard

Files and file descriptors
File descriptors allow efficient small consecutive reads or writes from or to a given file. The argument n below is always a descriptor, attached to a file in **r**(ead), **w**(rite) or **a**(ppend) mode.
get descriptor n for file $path$ in given $mode$
... from shell cmd output (pipe)
close descriptor
commit pending write operations
read logical line from file
... raw line from file
write $s \backslash n$ to file
... write s to file

forpart($p = n, seq$)
forperm(S, p, seq)
forsubset(n, p, seq)
forsubset($[n, k], p, seq$)
forqfvec(v, q, b, seq)
forsubgroup($H = G$)
until(a, seq)
while(a, seq)
break($\{n\}$)
next($\{n\}$)
return($\{x\}$)

error(), warning()
errname(E)
iferr(seq_1, E, seq_2)

arity(f)
select(f, v)
apply(f, v)
call(f, a)
fold(f, a)
self()

sum($X = a, b, expr, \{x\}$)
vecsum(v)
vecprod(v)
sumdiv($n, X, expr$)
sumdivmult($n, X, expr$)
prod($X = a, b, expr, \{x\}$)
prodeuler($X = a, b, expr$)

vecsort($x, \{k\}, \{fl = 0\}$)
vecmin($x, \{\&i\}$), vecmax
vecsearch($x, y, \{f\}$)
matreduce(m)

print, print1, printtex
printp
printsep($sep, \dots$), printsep1
printf()
write, write1, writetex($file, args$)
writebin($file, x$)
read($\{file\}$)
readvec($\{file\}$)
readstr($\{file\}$)
input()

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Timers

CPU time in ms and reset timer
CPU time in ms since gp startup
time in ms since UNIX Epoch
timeout command after s seconds

Interface with system

allocates a new stack of s bytes
alias old to new
install function from library
execute system command a
... and feed result to GP
... returning GP string
get \$VAR from environment
expand env. variable in string

Parallel evaluation

These functions evaluate their arguments in parallel (pthreads or MPI); args. must not access global variables (use export for this) and must be free of side effects. Enabled if threading engine is not *single* in gp header.

evaluate f on $x[1], \dots, x[n]$
evaluate closures $f[1], \dots, f[n]$
as select
as sum
as vector
eval f for $i = a, \dots, b$
... for each element x in v
... for p prime in $[a, b]$
... for $p = a \bmod q$
... multivariate
export x to parallel world
... all dynamic variables
frees exported value x
... all exported values

Linear Algebra

dimensions of matrix x
multiply two matrices
... assuming result is diagonal
concatenation of x and y
extract components of x
transpose of vector or matrix x
adjoint of the matrix x
eigenvectors/values of matrix x
characteristic/minimal polynomial of x
trace/determinant of matrix x
permanent of matrix x
Frobenius form of x
QR decomposition
apply matqr's transform to v

Constructors & Special Matrices

$\{g(x): x \in v \text{ s.t. } f(x)\}$
 $\{x: x \in v \text{ s.t. } f(x)\}$
 $\{g(x): x \in v\}$
row vec. of $expr$ eval'ed at $1 \leq i \leq n$
col. vec. of $expr$ eval'ed at $1 \leq i \leq n$
vector of small ints

gettime()
getabstime()
getwalltime()
alarm($s, expr$)
allocatemem($\{s\}$)
alias(new, old)
install($f, code, \{gpf\}, \{lib\}$)
system(a)
extern(a)
externstr(a)
getenv("VAR")
strexpand(x)

parapply(f, x)
pareval(f)
parselect($f, A, \{flag\}$)
parsum($i = a, b, expr$)
parvector($n, i, \{expr\}$)
parfor($i = a, \{b\}, f, \{r\}, \{f_2\}$)
parforeach($v, x, f, \{r\}, \{f_2\}$)
parforprime($p = a, \{b\}, f, \{r\}, \{f_2\}$)
parforprimestep($p = a, \{b\}, q, f, \{r\}, \{f_2\}$)
parforvec($X = v, f, \{r\}, \{f_2\}, \{flag\}$)
export(x)
exportall()
unexport(x)
unexportall()

matsize(x)
 $x * y$
matmultodiagonal(x, y)
concat($x, \{y\}$)
vecextract($x, y, \{z\}$)
 $x \sim$, mattranspose(x)
matadjoint(x)
mateigen(x)
charpoly(x), minpoly(x)
trace(x), matdet(x)
matpermanent(x)
matfrobenius(x)
matqr(x)
mathouseholder(Q, v)

$[g(x) \mid x \leftarrow v, f(x)]$
 $[x \mid x \leftarrow v, f(x)]$
 $[g(x) \mid x \leftarrow v]$
vector($n, \{i\}, \{expr\}$)
vectorv($n, \{i\}, \{expr\}$)
vectorsmall($n, \{i\}, \{expr\}$)

$[c, c \cdot x, \dots, c \cdot x^n]$
 $[1, 2^x, \dots, n^x]$
matrix $1 \leq i \leq m, 1 \leq j \leq n$
define matrix by blocks
diagonal matrix with diagonal x
is x diagonal?
 $x \cdot \text{matdiagonal}(d)$
 $n \times n$ identity matrix
Hessenberg form of square matrix x
 $n \times n$ Hilbert matrix $H_{ij} = (i + j - 1)^{-1}$
 $n \times n$ Pascal triangle
companion matrix to polynomial x
Sylvester matrix of x and y

Gaussian elimination

kernel of matrix x
intersection of column spaces of x and y
solve $MX = B$ (M invertible)
one sol of $M * X = B$
basis for image of matrix x
columns of x not in **matimage**
supplement columns of x to get basis
rows, cols to extract invertible matrix
rank of the matrix x
solve $MX = B \bmod D$
image mod D
kernel mod D
inverse mod D
determinant mod D

Lattices & Quadratic Forms

Quadratic forms

evaluate ${}^t x Q y$
evaluate ${}^t x Q x$
signature of quad form ${}^t y * x * y$
decomp into squares of ${}^t y * x * y$
eigenvalues/vectors for real symmetric x

HNF and SNF

upper triangular Hermite Normal Form
HNF of x where d is a multiple of $\det(x)$
multiple of $\det(x)$
HNF of $(x \mid \text{diagonal}(D))$
elementary divisors of x
 q -rank from elementary divisors
elementary divisors of $\mathbf{Z}[a]/(f'(a))$
integer kernel of x
 $\mathbf{Z}$ -module $\leftrightarrow$ $\mathbf{Q}$ -vector space

Lattices

LLL-algorithm applied to columns of x
... for Gram matrix of lattice
find up to m sols of $\mathbf{qfnorm}(x, y) \leq b$
 $v, v[i] :=$ number of y s.t. $\mathbf{qfnorm}(x, y) = i$
perfection rank of x
find isomorphism between q and Q
precompute for isomorphism test with q
automorphism group of q

Based on an earlier version by Joseph H. Silverman
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convert **qfauto** for GAP/Magma **qfautoexport**($G, \{flag\}$)
orbits of V under $G \subset \mathrm{GL}(V)$ **qforbits**(G, V)

Polynomials & Rational Functions

all defined polynomial variables **variables**()
get var. of highest priority (higher than v) **varhigher**($name, \{v\}$)
... of lowest priority (lower than v) **varlower**($name, \{v\}$)

Coefficients, variables and basic operators

degree of f **poldegree**(f)
coef. of degree n of f , leading coef. **polcoef**(f, n), **pollead**
main variable / all variables in f **variable**(f), **variables**(f)
replace x by y in f **subst**(f, x, y)
evaluate f replacing vars by their value **eval**(f)
replace polynomial expr. $T(x)$ by y in f **substpol**(f, T, y)
replace $x_1, \dots, x_n$ by $y_1, \dots, y_n$ in f **substvec**(f, x, y)

$f \in A[x]$; reciprocal polynomial $x^{\deg f} f\left(\frac{1}{x}\right)$ **polrecip**(f)
gcd of coefficients of f **content**(f)
derivative of f w.r.t. x **deriv**($f, \{x\}$)
... n -th derivative of f **derivn**($f, n, \{x\}$)
formal integral of f w.r.t. x **intformal**($f, \{x\}$)
formal sum of f w.r.t. x **sumformal**($f, \{x\}$)

Constructors & Special Polynomials

interpolation polynomial at $(x[1], y[1]), \dots, (x[n], y[n])$, evaluated at t , with error estimate e **polinterpolate**($x, \{y\}, \{t\}, \{&e\}$)
 $T_n/U_n, H_n$ **polchebyshev**(n), **polhermite**(n)
 $P_n, L_n^{(\alpha)}$ **pollegendre**(n), **pollaguerre**(n, a)
 n -th cyclotomic polynomial Φ_n **polcyclo**(n)
return n if $f = \Phi_n$, else 0 **poliscyclo**(f)
is f a product of cyclotomic polynomials? **poliscycloprod**(f)
Zagier's polynomial of index (n, m) **polzagier**(n, m)

Resultant, elimination

discriminant of polynomial f **poldisc**(f)
find factors of **poldisc**(f) **poldiscfactors**(f)
resultant $R = \mathrm{Res}_v(f, g)$ **polresultant**($f, g, \{v\}$)
 $[u, v, R], xu + yv = \mathrm{Res}_v(f, g)$ **polresultanttext**($x, y, \{v\}$)
solve Thue equation $f(x, y) = a$ **thue**($t, a, \{sol\}$)
initialize t for Thue equation solver **thueinit**(f)

Roots and Factorization (Complex/Real)

complex roots of f **polroots**(f)
bound complex roots of f **polrootsbound**(f)
number of real roots of f (in $[a, b]$) **polsturm**($f, \{[a, b]\}$)
real roots of f (in $[a, b]$) **polrootsreal**($f, \{[a, b]\}$)
complex embeddings of **t_POLMOD** z **conjsvec**(z)

Roots and Factorization (Finite fields)

factor f mod p , roots **factormod**(f, p), **polrootsmod**
factor f over $\mathbf{F}_p[x]/(T)$, roots **factormod**($f, [T, p]$), **polrootsmod**
squarefree factorization of f in $\mathbf{F}_q[x]$ **factormodSQF**($f, \{D\}$)
distinct degree factorization of f in $\mathbf{F}_q[x]$ **factormodDDF**($f, \{D\}$)
factor n -th cyclotomic pol. Φ_n mod p **factormodcyclo**(n, p)

Roots and Factorization (p -adic fields)

factor f over $\mathbf{Q}_p$, roots **factorpadic**(f, p, r), **polrootspadic**
 p -adic root of f congruent to a mod p **padicappr**(f, a)
Newton polygon of f for prime p **newtonpoly**(f, p)
Hensel lift $A/\mathrm{lc}(A) = \prod_i B[i] \bmod p^e$ **polhensellift**(A, B, p, e)
 $T = \prod (x - z_i) \mapsto \prod [x - \omega(z_i)] \in \mathbf{Z}_p[x]$ **polteichmuller**(T, p, e)
extensions of $\mathbf{Q}_p$ of degree N **padicfields**(p, N)

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Roots and Factorization (Miscellaneous)

symmetric powers of roots of f up to n **polsym**(f, n)
Graeffe transform of $f, g(x^2) = f(x)f(-x)$ **polgraeffe**(f)
factor f over coefficient field **factor**(f)
cyclotomic factors of $f \in \mathbf{Q}[X]$ **polcyclofactors**(f)

Finite Fields

A finite field is encoded by any element (**t_FFELT**).
find irreducible $T \in \mathbf{F}_p[x]$, $\deg T = n$ **ffinit**($p, n, \{x\}$)
Create t in $\mathbf{F}_q \simeq \mathbf{F}_p[t]/(T)$ **t = ffg****en**($T, 't$)
... indirectly, with implicit T **t = ffg****en**($q, 't$); **T = t.mod**
map m from $\mathbf{F}_q \ni a$ to $\mathbf{F}_{q^k} \ni b$ **m = ffg****em**(a, b)
build $K = \mathbf{F}_q[x]/(P)$ extending $\mathbf{F}_q \ni a$, **ffextend**(a, P)
evaluate map m on x **ffmap**(m, x)
inverse map of m **ffinvmap**(m)
compose maps $m \circ n$ **ffcompomap**(m, n)
 x as polmod over codomain of map m **ffmaprel**(m, x)
 F^n over $\mathbf{F}_q \ni a$ **fffrobenius**(a, n)
 $\#$ {monic irred. $T \in \mathbf{F}_q[x]$, $\deg T = n$ } **ffnbirred**(q, n)

Formal & p -adic Series

truncate power series or p -adic number **truncate**(x)
valuation of x at p **valuation**(x, p)
Dirichlet and Power Series
Taylor expansion around 0 of f w.r.t. x **taylor**(f, x)
Laurent series of closure F up to x^k **laurentseries**(f, k)
 $\sum a_k b_k t^k$ from $\sum a_k t^k$ and $\sum b_k t^k$ **serconvol**(a, b)
 $f = \sum a_k t^k$ from $\sum (a_k/k!) t^k$ **serlaplace**(f)
reverse power series F so $F(f(x)) = x$ **serreverse**(f)
remove terms of degree $< n$ in f **serchop**(f, n)
Dirichlet series multiplication / division **dirmul**, **dirdiv**(x, y)
Dirichlet Euler product (b terms) **direuler**($p = a, b, expr$)

Transcendental and p -adic Functions

real, imaginary part of x **real**(x), **imag**(x)
absolute value, argument of x **abs**(x), **arg**(x)
square/ n th root of x **sqrt**(x), **sqrtn**($x, n, \{&z\}$)
all n -th roots of 1 **rootsof1**(n)
FFT of $[f_0, \dots, f_{n-1}]$ **w = fft****init**(n), **fft/fftinv**(w, f)
trig functions **sin**, **cos**, **tan**, **cotan**, **sinc**
inverse trig functions **asin**, **acos**, **atan**
hyperbolic functions **sinh**, **cosh**, **tanh**, **cotanh**
inverse hyperbolic functions **asinh**, **acosh**, **atanh**
 $\log(x)$, $\log(1+x)$, e^x , $e^x - 1$ **log**, **log1p**, **exp**, **expm1**
Euler Γ function, $\log \Gamma$, Γ'/Γ **gamma**, **lngamma**, **psi**
half-integer gamma function $\Gamma(n+1/2)$ **gammah**(n)
Riemann's zeta $\zeta(s) = \sum n^{-s}$ **zeta**(s)
 $\sum_{1 \leq n \leq N} n^s$ **dirpowerssum**(N, s)
Hurwitz's $\zeta(s, x) = \sum (n+x)^{-s}$ **zetahurwitz**(s, x)
Lerch $\Phi(z, s, x) = \sum z^n (n+x)^{-s}$ **lerchphi**(z, s, x)
Lerch $L(s, x, t) = \Phi(e^{2i\pi t}, s, x)$ **lerchzeta**(s, x, t)
multiple zeta value (MZV), $\zeta(s_1, \dots, s_k)$ **zetamult**($s, \{T\}$)
all MZVs for weight $\sum s_i = n$ **zetamultall**(n)
convert MZV id to $[s_1, \dots, s_k]$ **zetamultconvert**($f, \{flag\}$)
MZV dual sequence **zetamultdual**(s)
multiple polylog $Li_{s_1, \dots, s_k}(z_1, \dots, z_k)$ **polylogmult**(s, z)

incomplete Γ function ($y = \Gamma(s)$) **incgam**($s, x, \{y\}$)
complementary incomplete Γ **incgamc**(s, x)
 $\int_x^\infty e^{-t} dt/t$, $(2/\sqrt{\pi}) \int_x^\infty e^{-t^2} dt$ **eint1**, **erfc**
elliptic integral of 1st and 2nd kind **ellK**(k), **ellE**(k)
dilogarithm of x **dilog**(x)
 m -th polylogarithm of x **polylog**($m, x, \{flag\}$)
 U -confluent hypergeometric function **hyperu**(a, b, u)
Hypergeometric ${}_pF_q(A, B; z)$ **hypergeom**(A, B, z)
Bessel $J_n(x)$, $J_{n+1/2}(x)$ **besselj**(n, x), **besseljh**(n, x)
Bessel $I_\nu, K_\nu, H_\nu^1, H_\nu^2, Y_\nu$ (**bessel**)**i**, **k**, **h1**, **h2**, **y**
 k -th zero of $J_\nu(x)$ **besseljzero**($nu, \{k=1\}$)
 k -th zero of $Y_\nu(x)$ **besselyzero**($nu, \{k=1\}$)
Airy functions $A_i(x)$, $B_i(x)$ **airy**(x)
Lambert W : x s.t. $xe^x = y$ **lambertw**(y)
Teichmuller character of p -adic x **teichmuller**(x)

Iterations, Sums & Products

Numerical integration for meromorphic functions

Behaviour at endpoint for Double Exponential (DE) methods: either a scalar ($a \in \mathbf{C}$, regular) or $\pm\infty$ (decreasing at least as x^{-2}) or
 $(x-a)^{-\alpha}$ singularity $[a, a]$
exponential decrease $e^{-\alpha|x|}$ $[\pm\infty, a]$, $\alpha > 0$
slow decrease $|x|^\alpha$ $\dots \alpha < -1$
oscillating as $\cos(kx)$ $\alpha = k\mathbf{I}$, $k > 0$
oscillating as $\sin(kx)$ $\alpha = -k\mathbf{I}$, $k > 0$

numerical integration **intnum**($x = a, b, f, \{T\}$)
weights T for **intnum** **intnuminit**($a, b, \{m\}$)
weights T incl. kernel K **intfuncinit**($t = a, b, K, \{m\}$)
integrate $(2i\pi)^{-1} f$ on circle $|z-a| = R$ **intcirc**($x = a, R, f, \{T\}$)
Other integration methods
 n -point Gauss-Legendre **intnumgauss**($x = a, b, f, \{n\}$)
weights for n -point Gauss-Legendre **intnumgaussinit**($\{n\}$)
quasi-periodic function, period $2H$ **intnumosc**($x = a, f, H$)
Romberg (low accuracy) **intnumromb**($x = a, b, f, \{flag\}$)

Numerical summation

sum of series $f(n)$, $n \geq a$ (low accuracy) **suminf**($n = a, expr$)
sum of alternating/positive series **sumalt**, **sumpos**
sum of series using Euler-Maclaurin **sumnum**($n = a, f, \{T\}$)
... Sidi summation **sumnumsidi**($n = a, f$)
 $\sum_{n \geq a} F(n)$, F rational function **sumnumrat**(F, a)
 $\dots \sum_{p \geq a} F(p^s)$ **sumeulerrat**($F, \{s=1\}, \{a=2\}$)
weights for **sumnum**, a as in DE **sumnuminit**($\{\infty, a\}$)
sum of series by Monien summation **sumnummonien**($n = a, f, \{T\}$)
weights for **sumnummonien** **sumnummonieninit**($\{\infty, a\}$)
sum of series using Abel-Plana **sumnumap**($n = a, f, \{T\}$)
weights for **sumnumap**, a as in DE **sumnumapinit**($\{\infty, a\}$)
sum of series using Lagrange **sumnumlagrange**($n = a, f, \{T\}$)
weights for **sumnumlagrange** **sumnumlagrangeinit**

Products

product $a \leq X \leq b$, initialized at x **prod**($X = a, b, expr, \{x\}$)
product over primes $a \leq X \leq b$ **prodeuler**($X = a, b, expr$)
infinite product $a \leq X \leq \infty$ **prodinf**($X = a, expr$)
 $\prod_{n \geq a} F(n)$, F rational function **prodnumrat**(F, a)
 $\prod_{p \geq a} F(p^s)$ **prodeulerrat**($F, \{s=1\}, \{a=2\}$)

Other numerical methods

real root of f in $[a, b]$; bracketed root	<code>solve($X = a, b, f$)</code>
...interval splitting, step s	<code>solvestep($X = a, b, s, f, \{flag = 0\}$)</code>
limit of $f(t)$, $t \rightarrow \infty$	<code>limitnum($f, \{\alpha\}$)</code>
asymptotic expansion of f (rational)	<code>asypnum($f, \{\alpha\}$)</code>
... $N + 1$ terms as floats	<code>asypnumraw($f, N, \{\alpha\}$)</code>
numerical derivation w.r.t x : $f'(a)$	<code>derivnum($x = a, f$)</code>
evaluate continued fraction F at t	<code>contfraceval($F, t, \{L\}$)</code>
power series to cont. fraction (L terms)	<code>contfracinit($S, \{L\}$)</code>
Padé approximant (deg. denom. $\leq B$)	<code>bestapprPade($S, \{B\}$)</code>

Elementary Arithmetic Functions

vector of binary digits of $ x $	<code>binary(x)</code>
bit number n of integer x	<code>bittest(x, n)</code>
Hamming weight of integer x	<code>hammingweight(x)</code>
digits of integer x in base B	<code>digits($x, \{B = 10\}$)</code>
sum of digits of integer x in base B	<code>sumdigits($x, \{B = 10\}$)</code>
integer from digits	<code>fromdigits($v, \{B = 10\}$)</code>
ceiling/floor/fractional part	<code>ceil, floor, frac</code>
round x to nearest integer	<code>round($x, \{\&e\}$)</code>
truncate x	<code>truncate($x, \{\&e\}$)</code>
gcd/LCM of x and y	<code>gcd(x, y), lcm(x, y)</code>
gcd of entries of a vector/matrix	<code>content(x)</code>

Primes and Factorization

extra prime table	<code>addprimes()</code>
add primes in v to prime table	<code>addprimes(v)</code>
remove primes from prime table	<code>removeprimes(v)</code>
Chebyshev $\pi(x)$, n -th prime p_n	<code>primepi(x), prime(n)</code>
vector of first n primes	<code>primes(n)</code>
smallest prime $\geq x$	<code>nextprime(x)</code>
largest prime $\leq x$	<code>precprime(x)</code>
factorization of x	<code>factor($x, \{lim\}$)</code>
...selecting specific algorithms	<code>factorint($x, \{flag = 0\}$)</code>
$n = df^2$, d squarefree/fundamental	<code>core($n, \{fl\}$), coredisc</code>
certificate for (prime) N	<code>primecert(N)</code>
verifies a certificate c	<code>primecertisvalid(c)</code>
convert certificate to Magma/PRIMO	<code>primecertexport</code>
recover x from its factorization	<code>factorback($f, \{e\}$)</code>
$x \in \mathbf{Z}$, $ x \leq X$, $\gcd(N, P(x)) \geq N$	<code>zncoppersmith($P, N, X, \{B\}$)</code>
divisors of N in residue class r mod s	<code>divisorslensstra(N, r, s)</code>

Divisors and multiplicative functions

number of prime divisors $\omega(n)$ / $\Omega(n)$	<code>omega(n), bigomega</code>
divisors of n / number of divisors $\tau(n)$	<code>divisors(n), numdiv</code>
sum of (k -th powers of) divisors of n	<code>sigma($n, \{k\}$)</code>
Möbius μ -function	<code>moebius(x)</code>
Ramanujan's τ -function	<code>ramanujantau(x)</code>

Combinatorics

factorial of x	<code>x!</code> or <code>factorial(x)</code>
binomial coefficient $\binom{x}{k}$	<code>binomial($x, \{k\}$)</code>
Bernoulli number B_n as real/rational	<code>bernreal(n), bernfrac</code>
$[B_0, B_2, \dots B_{2k}]$	<code>bernvec(k)</code>
Bernoulli polynomial $B_n(x)$	<code>bernpol($n, \{x\}$)</code>
Euler numbers	<code>eulerfrac, eulerreal, eulervec</code>
Euler polynomial $E_n(x)$	<code>eulerpol($n, \{x\}$)</code>
Eulerian polynomial $A_n(x)$	<code>eulerianpol</code>
Fibonacci number F_n	<code>fibonacci(n)</code>
Harmonic number $H_{n,r} = 1^{-r} + \dots + n^{-r}$	<code>harmonic(n, r)</code>
Stirling numbers $s(n, k)$ and $S(n, k)$	<code>stirling($n, k, \{flag\}$)</code>

Pari-GP reference card

(PARI-GP version 2.15.3)

number of partitions of n	<code>numbpart(n)</code>
k -th permutation on n letters	<code>numtoperm(n, k)</code>
...index k of permutation v	<code>permtomv(n)</code>
order of permutation p	<code>permorder(p)</code>
signature of permutation p	<code>permsign(p)</code>
cyclic decomposition of permutation p	<code>permcycles(p)</code>

Multiplicative groups $(\mathbf{Z}/N\mathbf{Z})^*$, $\mathbf{F}_q^*$

Euler ϕ -function	<code>eulerphi(x)</code>
multiplicative order of x (divides ϕ)	<code>znorder($x, \{o\}$), fforder</code>
primitive root mod q / x .mod	<code>znprimroot(q), fprimroot(x)</code>
structure of $(\mathbf{Z}/n\mathbf{Z})^*$	<code>znstar(n)</code>
discrete logarithm of x in base g	<code>znlog($x, g, \{o\}$), fflag</code>
Kronecker-Legendre symbol $(\frac{x}{y})$	<code>kronecker(x, y)</code>
quadratic Hilbert symbol (at p)	<code>hilbert($x, y, \{p\}$)</code>

Euclidean algorithm, continued fractions

CRT: solve $z \equiv x$ and $z \equiv y$	<code>chinese(x, y)</code>
minimal u, v so $xu + yv = \gcd(x, y)$	<code>gcdext(x, y)</code>
half-gcd algorithm	<code>halfgcd(x, y)</code>
continued fraction of x	<code>confrac($x, \{b\}, \{lmax\}$)</code>
last convergent of continued fraction x	<code>confracpnqn(x)</code>
rational approximation to x (den. $\leq B$)	<code>bestappr($x, \{B\}$)</code>
recognize $x \in \mathbf{C}$ as polmod mod $T \in \mathbf{Z}[X]$	<code>bestapprnf(x, T)</code>

Miscellaneous

integer square / n -th root of x	<code>sqrtint(x), sqtrtnint(x, n)</code>
largest integer e s.t. $b^e \leq b$, $e = \lfloor \log_b(x) \rfloor$	<code>logint($x, b, \{\&z\}$)</code>

Characters

Let $cyc = [d_1, \dots, d_k]$ represent an abelian group $G = \oplus (\mathbf{Z}/d_j\mathbf{Z}) \cdot g_j$ or any structure G affording a .cyc method; e.g. `znstar( $q, 1$ )` for Dirichlet characters. A character χ is coded by $[c_1, \dots, c_k]$ such that $\chi(g_j) = e(n_j/d_j)$.
 $\chi \cdot \psi$; χ^{-1} ; $\chi \cdot \psi^{-1}$; χ^k `charmul, charconj, chardiv, charpow`
order of χ `charorder( $cyc, \chi$ )`
kernel of χ `charker( $cyc, \chi$ )`
 $\chi(x)$, G a GP group structure `chareval( $G, \chi, x, \{z\}$ )`
Galois orbits of characters `chargalois( $G$ )`

Dirichlet Characters

initialize $G = (\mathbf{Z}/q\mathbf{Z})^*$	<code>G = znstar($q, 1$)</code>
convert datum D to $[G, \chi]$	<code>znchar(D)</code>
is χ odd?	<code>zncharisodd(G, χ)</code>
real $\chi \rightarrow$ Kronecker symbol $(D/.)$	<code>znchartokronecker(G, χ)</code>
conductor of χ	<code>zncharconductor(G, χ)</code>
$[G_0, \chi_0]$ primitive attached to χ	<code>znchartoprimitive(G, χ)</code>
induce $\chi \in \hat{G}$ to $\mathbf{Z}/N\mathbf{Z}$	<code>zncharinduce(G, χ, N)</code>
χp	<code>znchardecompose(G, χ, p)</code>
$\prod_p (\chi, N) \chi p$	<code>znchardecompose(G, χ, Q)</code>
complex Gauss sum $G_a(\chi)$	<code>znchargauss(G, χ)</code>

Conrey labelling

Conrey label $m \in (\mathbf{Z}/q\mathbf{Z})^* \rightarrow$ character	<code>znconreychar(G, m)</code>
character $\rightarrow$ Conrey label	<code>znconreyexp(G, χ)</code>
log on Conrey generators	<code>znconreylog(G, m)</code>
conductor of χ (χ_0 primitive)	<code>znconreyconductor($G, \chi, \{\chi_0\}$)</code>

True-False Tests

is x the disc. of a quadratic field?	<code>isfundamental(x)</code>
is x a prime?	<code>isprime(x)</code>
is x a strong pseudo-prime?	<code>ispseudoprime(x)</code>
is x square-free?	<code>issquarefree(x)</code>
is x a square?	<code>issquare($x, \{\&n\}$)</code>
is x a perfect power?	<code>ispower($x, \{k\}, \{\&n\}$)</code>
is x a perfect power of a prime? ($x = p^n$)	<code>isprimepower($x, \&n$)</code>
... of a pseudoprime?	<code>ispseudoprimepower($x, \&n$)</code>
is x powerful?	<code>ispowerful(x)</code>
is x a totient? ($x = \varphi(n)$)	<code>istotient($x, \{\&n\}$)</code>
is x a polygonal number? ($x = P(s, n)$)	<code>ispolygonal($x, s, \{\&n\}$)</code>
is pol irreducible?	<code>polisirreducible(pol)</code>

Graphic Functions

crude graph of $expr$ between a and b	<code>plot($X = a, b, expr$)</code>
High-resolution plot (immediate plot)	<code>plot(X = a, b, expr, {flag}, {n})</code>
plot $expr$ between a and b	<code>ploth($X = a, b, expr, \{flag\}, \{n\}$)</code>
plot points given by lists lx, ly	<code>plothraw($lx, ly, \{flag\}$)</code>
terminal dimensions	<code>plotsizes()</code>
Rectwindow functions	
init window w , with size x, y	<code>plotinit(w, x, y)</code>
erase window w	<code>plotkill(w)</code>
copy w to w_2 with offset (dx, dy)	<code>plotcopy(w, w_2, dx, dy)</code>
slice contents of w	<code>plotclip(w)</code>
scale coordinates in w	<code>plotscale(w, x_1, x_2, y_1, y_2)</code>
ploth in w	<code>plotrecth($w, X = a, b, expr, \{flag\}, \{n\}$)</code>
plothraw in w	<code>plotrecthraw($w, data, \{flag\}$)</code>
draw window w_1 at $(x_1, y_1), \dots$	<code>plotdraw([$w_1, x_1, y_1], \dots$)</code>

Low-level Rectwindow Functions

set current drawing color in w to c	<code>plotcolor(w, c)</code>
current position of cursor in w	<code>plotcursor(w)</code>
write s at cursor's position	<code>plotstring(w, s)</code>
move cursor to (x, y)	<code>plotmove(w, x, y)</code>
move cursor to $(x + dx, y + dy)$	<code>plotrmove(w, dx, dy)</code>
draw a box to (x_2, y_2)	<code>plotbox(w, x_2, y_2)</code>
draw a box to $(x + dx, y + dy)$	<code>plotrbox(w, dx, dy)</code>
draw polygon	<code>plotlines($w, lx, ly, \{flag\}$)</code>
draw points	<code>plotpoints(w, lx, ly)</code>
draw line to $(x + dx, y + dy)$	<code>plotrline(w, dx, dy)</code>
draw point $(x + dx, y + dy)$	<code>plotrpoint(w, dx, dy)</code>

Convert to Postscript or Scalable Vector Graphics

The format f is either "ps" or "svg".	
as ploth	<code>plotlexport($f, X = a, b, expr, \{flag\}, \{n\}$)</code>
as plothraw	<code>plotlrawlexport($f, lx, ly, \{flag\}$)</code>
as plotdraw	<code>plotlexport($f, [[w_1, x_1, y_1], \dots]$)</code>

Based on an earlier version by Joseph H. Silverman
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